Exact Calculation of Scattering Parameters of the Coplanar–Slot Transition in Unilateral Finline Technology

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Abstract —**The dominant and higher order modes, in both a unilateral finline and coupled unilateral finlines in the even and odd modes, are accurately described from a thorough spectral-domain approach. Then, coupling coefficients between eigenmodes at a coplanar–slot transition in unilateral finline technology, which are to be used in the generalized scattering matrix formulation, are directly computed in the spectral domain. Scattering parameters of the dominant mode in the *Ka*-band are presented for both even- and odd-mode excitation of the coupled unilateral finlines.**

I. Introduction

THE E-PLANE coplanar–slot transition like that shown in Fig. 1 forms a frequency-independent 180° hybrid, serving as the basis for the design of broad-band planar microwave and millimeter balanced mixers [1], [2]. A complete review of these numerous realizations reveals the nonexistence of any rigorous electromagnetic analysis for this transition that can serve as a solid basis for its design.

It is clear that an odd-mode excitation for the coupled unilateral finlines (coplanar line function), by ensuring that the outer conductors have the same potential along the whole length, creates a balanced–unbalanced (balun) junction and hence its two arms are essentially decoupled. This is technically impossible and both the even and odd modes are often excited and are coupled to the waves that can be excited on the other side of the junction. These are mainly even ones with respect to the x axis, but there is a possibility of also exciting odd evanescent modes.

Therefore, the aim of this paper is twofold: i) to give an accurate evaluation of several eigenmodes for coupled unilateral finlines in the even- and odd-mode excitations and of some evanescent odd eigenmodes for the unilateral finline, by means of a thorough spectral-domain approach (to our knowledge, there are no previous results on this subjects) and ii) to determine the generalized scattering

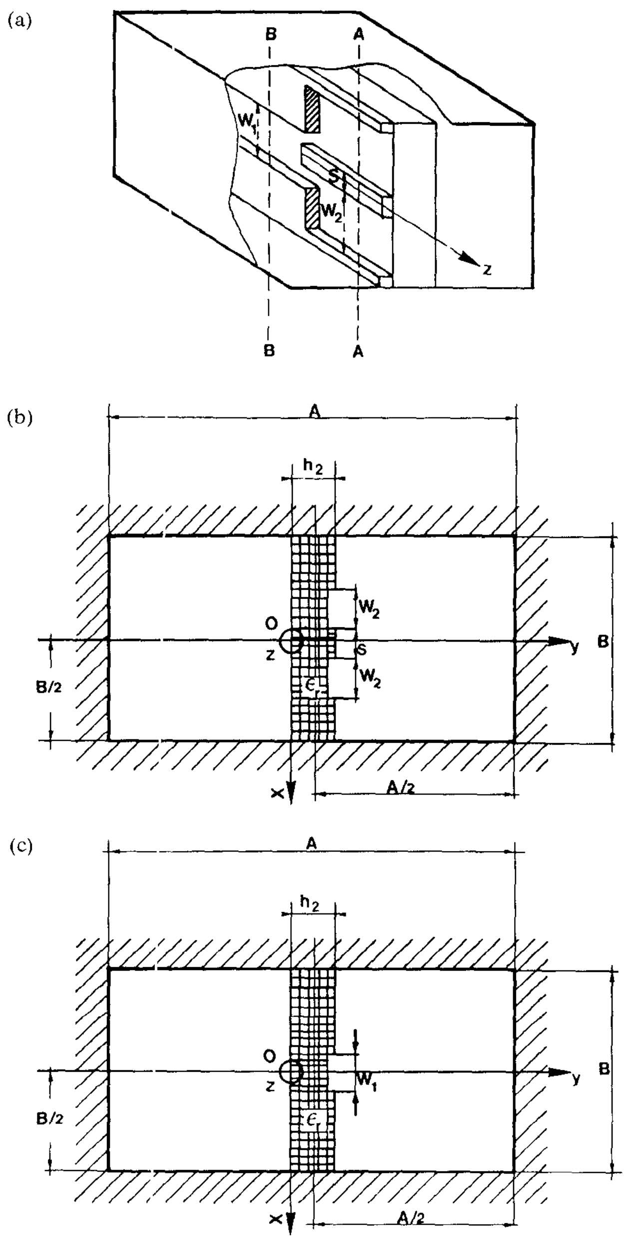

Fig. 1 (a) The E-plane coplanar–slot transition. (b) $A-A$ cross section (unilateral coupled lines). (c) $B-B$ cross section (unilateral finline). Here $A = 7.112$ mm, $B = 3.556$ mm, $h_2 = 0.254$ mm, and $\epsilon_r = 2.22$.

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matrix for the E-plane coplanar–slot transition by combining the direct modal analysis [3] and the spectral-domain approach taking into consideration all possible mode parities.

II. Evaluation of Eigenmodes for Finlines Forming the Transition

The axial field components $E_{z,i}(x, y)$ and $H_{z,i}(x, y)$ in the ith region ($i = 1,2,3$) are expanded in Fourier series (Fig. 1(b) and (c)) within their domain

$$-B/2 \leqslant x \leqslant B/2.$$

The following expansions of $E_{z,i}(x, y)$ and $H_{z,i}(x, y)$ are valid:

$$E_{z,i}(x, y) = \sum_{m=1}^{\infty} \tilde{E}_{z,i}(m, y) \sin(\alpha_m x)$$

$$H_{z,i}(x, y) = \sum_{m=0}^{\infty} \tilde{H}_{z,i}(m, y) \cos(\alpha_m x) \tag{1}$$

where $\alpha_m = 2m\pi/B$ for even modes and $\alpha_m = (2m-1)\pi/B$ for odd modes and where quantities with the sign $\sim$ designate the line amplitude (i.e., the mth term of the Fourier series) associated with the space harmonic α_m.

The partial differential equations for the axial field components $E_{z,i}(x, y)$ and $H_{z,i}(x, y)$ are also Fourier expanded with respect to x; ordinary differential equations are derived for the mth line amplitudes $\tilde{E}_{z,i}(m, y)$ and $\tilde{H}_{z,i}(m, y)$, respectively.

For the studied structures (Fig. 1), these mth line amplitudes are given in [4, appendix I].

Through the application of boundary conditions at $y = 0$ and $y = h_2$, which are also Fourier expanded, the spectral coefficients are related to each other, to the mth line amplitudes of fin surface components denoted $\tilde{J}_x(m, h_2)$ and $\tilde{J}_z(m, h_2)$, and to the mth line amplitude of the slot aperture field components denoted $\tilde{E}_x(m, h_2)$ and $\tilde{E}_z(m, h_2)$. Extensive algebraic manipulations of these boundary conditions then yield functional equations relating the mth line amplitude of the fin surface current components to the mth line amplitude of the slot aperture field components. For a unilateral finline and for coupled unilateral finlines, the standard computational scheme uses the following admittance matrix representations of these functional equations:

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} \tilde{E}_x(m, h_2) \\ \tilde{E}_z(m, h_2) \end{bmatrix} = \begin{bmatrix} \tilde{J}_x(m, h_2) \\ \tilde{J}_z(m, h_2) \end{bmatrix}. \tag{2}$$

Closed-form expressions of the $\bar{\bar{G}}$ matrix elements are listed in [4, appendix II].

The numerical part is started by applying Galerkin's form of the general method of moments to the functional equations (2). The slot aperture field components $E_x(x, h_2)$ and $E_z(x, h_2)$ are expanded in terms of two complete sets of R and S basis functions, denoted $\mathscr{E}_{x,r}(x, h_2)$ ($r = 1, \cdots, R$) and $\mathscr{E}_{z,s}(x, h_2)$ ($s = 1, \cdots, S$):

$$E_x(x, h_2) = \sum_{r=1}^{R} a_r \mathscr{E}_{x,r}(x, h_2)$$

$$E_z(x, h_2) = \sum_{s=1}^{S} b_s \mathscr{E}_{z,s}(x, h_2). \tag{3}$$

Obviously, for the unilateral finline, the basis functions in (3) have nonzero values in the interval $-W/2 \leqslant x \leqslant W/2$ of the cell $-B/2 \leqslant x \leqslant B/2$, while for the coupled unilateral finline they have nonzero values in the two intervals $S/2 \leqslant x \leqslant (S/2) + W_2$ and $(-S/2) - W_2 \leqslant x \leqslant -S/2$.

By using an inner product consistent with Parseval's theorem, Galerkin's procedure is directly applied to the matrix form (2) in the Fourier domain. A set of $R + S$ homogeneous and linear equations, for which the $R + S$ unknowns are precisely the constants a_r and b_s, is obtained.

Nontrivial solutions of this set of equations occur for zero values of its matrix determinant. The real roots (i.e., $\beta^2 > 0$) determine the propagating eigenmodes, whereas the imaginary roots (i.e., $\beta^2 < 0$) determine evanescent ones.

Calculation of Basis Functions

The key point for an efficient eigenmode evaluation is the suitable choice of the set of basis functions into which the slot apertures fields are to be expanded. The basis functions utilized to describe both the dominant and higher order even modes for these fields in a unilateral finline were accurately determined and given in [4]. In Table I, we present the basis functions that we have utilized to describe the higher order fields of odd symmetry in the slot of a unilateral finline. Dispersion characteristics of the first four evanescent odd eigenmodes in a unilateral finline are plotted in Fig. 2. Note that they have the same nature as similar odd degenerate modes in a rectangular waveguide.

On the other hand, to describe both the dominant and the higher order modes of coupled unilateral finlines, the aperture fields, expanded into the basis functions shown in Table II for the even-mode excitation and in Table III for the odd-mode excitation (coplanar waveguide function), are appropriate for describing at least four eigenmodes.

Dispersion characteristics of four eigenmodes in coupled unilateral finlines in the even- and odd-mode excitations are plotted in Fig. 3 for the line parameter shown.

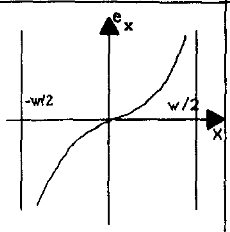
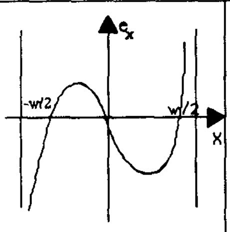
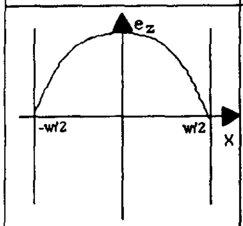
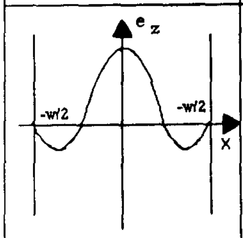
III. The E-Plane Coplanar-Slot Transition

The scattering matrix formulation of an axial waveguide discontinuity as derived from a direct modal analysis is described in detail in [4].

The transverse electric and magnetic field to the left and the right of the junction plane $z = 0$ can be expressed, respectively, as

$$\vec{E}_T^{\,\mathrm{I}} = \sum_{p=1}^{P} \left(A_p^{\mathrm{I}} + B_p^{\mathrm{I}}\right) \vec{e}_{p,T}^{\,\mathrm{I}} \qquad \vec{H}_T^{\,\mathrm{I}} = \sum_{p=1}^{P} \left(A_p^{\mathrm{I}} - B_p^{\mathrm{I}}\right) \vec{h}_{p,T} \tag{4}$$

TABLE I
BASIS FUNCTIONS AND ASSOCIATED mTH FOURIER COEFFICIENTS OF ODD MODES FOR A UNILATERAL FINLINE

	F(x)		$\tilde{f}(m)$
	$\lvert x\rvert \le w/2$	$e_x = \dfrac{T_1(2x/w)}{\sqrt{1-(2x/w)^2}}$	$\dfrac{j\pi w J_1(\alpha w/2)}{b}$
	$\lvert x\rvert \ge w/2$	0	
	$\lvert x\rvert \le w/2$	$e_x = \dfrac{T_3(2x/w)}{\sqrt{1-(2x/w)^2}}$	$\dfrac{j\pi w J_3(\alpha w/2)}{b}$
	$\lvert x\rvert \ge w/2$	0	
	$\lvert x\rvert \le w/2$	$e_z = \sqrt{1-(2x/w)^2}$	$\dfrac{\pi J_1(\alpha w/2)}{b\alpha}$
	$\lvert x\rvert \ge w/2$	0	
	$\lvert x\rvert \le w/2$	$e_z = \cos(3\pi x/w)$	$\dfrac{\pi \cos(\alpha w/2)}{bw\left[\left(\frac{3\pi}{w}\right)^2 - \alpha^2\right]}$
	$\lvert x\rvert \ge w/2$	0	

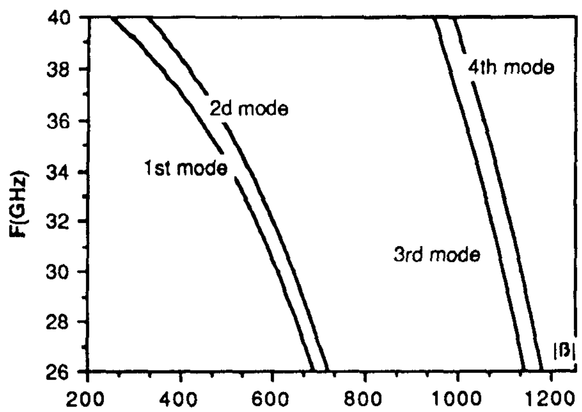


Fig. 2. Dispersion characteristics of four evanescent odd eigenmodes in unilateral finline.

and

$$\vec{E}_T^{\,\mathrm{II}} = \sum_{q=1}^{Q} \left(A_q^{\mathrm{II}} + B_q^{\mathrm{II}}\right) \vec{e}_{q,T}^{\,\mathrm{II}} \qquad \vec{H}_T^{\mathrm{II}} = \sum_{q=1}^{Q} \left(B_q^{\mathrm{II}} - A_q^{\mathrm{II}}\right) \vec{h}_{q,T}^{\,\mathrm{II}} \tag{5}$$

Here $\vec{e}_{p,T}, \vec{h}_{p,T}$ are normalized transverse electric and magnetic fields associated with the eigenmode p (respectively q) of the unilateral finline I (respectively coplanar line II), while the modal amplitudes A_p^{I}, B_p^{I} (respectively: A_q^{II}, B_q^{II}) refer to incident and reflected waves in the waveguide I (respectively II) at the junction plane.

Writing boundary conditions at the junction plane $z = 0$, transforming them into an equivalent set of linear equations involving $2(P+Q)$ modal amplitudes A_p^{I}, B_p^{I}, A_q^{II}, and B_q^{II} to be determined, the generalized scattering matrix of the junction can be constructed and put into a matrix form as

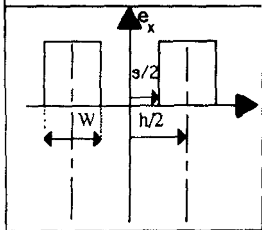
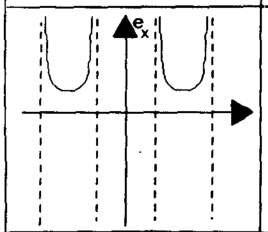
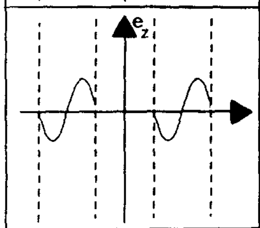
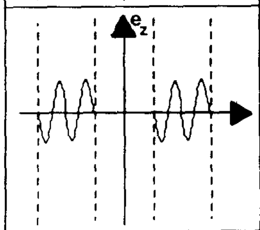
$$[B] = [S][A] \tag{6}$$

where

$$[B] = \begin{bmatrix} [B^{\mathrm{I}}] \\ [B^{\mathrm{II}}] \end{bmatrix} \qquad [A] = \begin{bmatrix} [A^{\mathrm{I}}] \\ [A^{\mathrm{II}}] \end{bmatrix}. \tag{7}$$

This generalized scattering matrix of the junction assembles both two reflection and two transmission matrix blocks. These matrix blocks have $P \times Q$ elements according to the numbers P and Q of eigenmodes that are used in the field expansion in waveguides at each side of the discontinuity plane $z = 0$ (see Fig. 1). During the derivation

TABLE II
BASIS FUNCTIONS AND ASSOCIATED mTH FOURIER COEFFICIENT OF EVEN MODES FOR COUPLED UNILATERAL FINLINES

	F(x)		$\tilde{f}(m)$
	± 1 0	$s/2<\|x\|<s/2+W$ $\|x\|<s/2$ $\|x\|<s/2+W$	$\frac{4}{B\alpha}\sin\alpha\frac{W}{2}\sin\alpha\frac{s+W}{2}$
	$\frac{\pm 1}{\sqrt{1-\left(\frac{2(\|x\|-h/2)}{w}\right)^2}}$ 0	$s/2<\|x\|<s/2+W$ $\|x\|<s/2$ $\|x\|<s/2+W$	$\frac{\pi W}{B} J_0\frac{\alpha W}{2}\sin\alpha\frac{s+W}{2}$
	$\left(\frac{h}{2}-\|x\|\right)\sqrt{1-\left(\frac{2(\|x\|-h/2)}{w}\right)^2}$ 0	$s/2<\|x\|<s/2+W$ $\|x\|<s/2$ $\|x\|<s/2+W$	$j\frac{\pi}{\alpha B}\sin\alpha\frac{s+W}{2}J_2\left(\alpha\frac{W}{2}\right)$
	$\frac{\sin(4\pi(\|x\|+h/2)/w)}{\sqrt{1-\left(\frac{2(\|x\|-h/2)}{w}\right)^2}}$ 0	$s/2<\|x\|<s/2+W$ $\|x\|<s/2$ $\|x\|<s/2+W$	$j\frac{\pi W}{2B}\sin\alpha\left(\frac{s+W}{2}\right)$ $[J_0(2\pi-\alpha W/2)-J_0(2\pi+\alpha W/2)]$

of matrix block elements, both coupling coefficients between eigenmodes at the discontinuity plane and power flow incident on it have been computed. By transforming them into the Fourier domain and using Parseval's theorem, the computation is made easier with the eigenmode spectral field components.

It can be proved easily that fields associated with modes of opposite parities on both sides of the junction cannot be coupled to each other. The possibility of coupling between i) even-mode fields and ii) odd-mode fields on both sides of the transition remains to be studied.

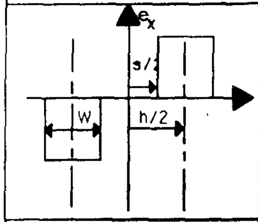
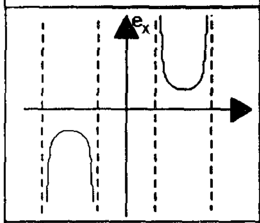
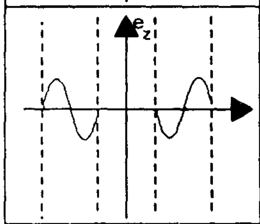
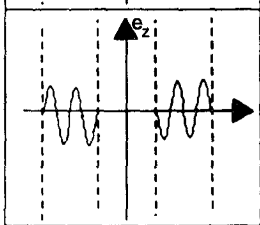
As an example, the E-plane coplanar–slot transition of Fig. 1 with $W_1 = 1$ mm, $W_2 = 0.95$ mm, and $S = 0.1$ mm in the Ka-band is analyzed.

The coupling between even modes excited on the unilateral finline and the coplanar line was studied first. The most critical factor in the modal analysis is the convergence of reflection and transmission coefficients as a function of the number of modes taken into consideration when writing the boundary conditions at the junction plane. After executing a number of systematic tests of convergence similar to those used in [4], the same number of eigenmodes was employed in each waveguide (i.e., $P = Q = 4$). Fig. 4 gives the module of calculated reflection and transmission coefficient for the transition in the case of even-mode excitation. It is interesting to add that the modules of S_{21} and S_{22} were nearly the same as those of S_{12} and S_{11} over the whole frequency band. The dashed lines in Fig. 4 represent the values of the scattering parameters for a step discontinuity in a unilateral finline technology having W_1 and $(2W_2 + S)$ as slot widths. The results reveal that both junctions behave in a similar way; the difference of the amplitudes yields quantitatively the effect of the central conductor in the coplanar side of the junction.

On the other hand, studying the coupling between four odd eigenmodes on the unilateral finline side (Fig. 2) and four similar ones on the coplanar side (Fig. 3(a)) shows that this coupling is very weak and that it hardly affects the characteristics of the transition. It is interesting to state that, in this case, the phase of the calculated reflection coefficient is found to be always 180° from the unilateral finline side and 0° from the coplanar side.

Fig. 5 shows results for another E-plane coplanar–slot transition operating in the frequency band 40–60 GHz with $W_1 = 0.75$ mm, $W_2 = 0.7$, mm and $S = 0.1$ mm.

TABLE III
BASIS FUNCTIONS AND ASSOCIATED mTH FOURIER COEFFICIENT OF ODD MODES FOR COUPLED UNILATERAL FINLINES

	F(x)		$\tilde{f}(m)$
	± 1	$s/2<\lvert x\rvert<s/2+w$	$\frac{4}{B\alpha}\sin\alpha\frac{w}{2}\sin\alpha\frac{s+w}{2}$
	0	$\lvert x\rvert<s/2$, $\lvert x\rvert>s/2+w$	
	$\frac{\pm 1}{\sqrt{1-\left(\frac{2(\lvert x\rvert-h/2)}{w}\right)^2}}$	$s/2<\lvert x\rvert<s/2+w$	$\frac{\pi W}{B}J_0\frac{\alpha W}{2}\sin\alpha\frac{s+W}{2}$
	0	$\lvert x\rvert<s/2$, $\lvert x\rvert>s/2+w$	
	$\left(\frac{h}{2}-\lvert x\rvert\right)\sqrt{1-\left(\frac{2(\lvert x\rvert-h/2)}{w}\right)^2}$	$s/2<\lvert x\rvert<s/2+w$	$j\frac{\pi}{\alpha B}\sin\alpha\frac{s+w}{2}J_2\left(\alpha\frac{w}{2}\right)$
	0	$\lvert x\rvert<s/2$, $\lvert x\rvert>s/2+w$	
	$\frac{\sin(4\pi(\lvert x\rvert+h/2)/w)}{\sqrt{1-\left(\frac{2(\lvert x\rvert-h/2)}{w}\right)^2}}$	$s/2<\lvert x\rvert<s/2+w$	$j\frac{\pi w}{2B}\sin\alpha\left(\frac{s+w}{2}\right)$ $[J_0(2\pi-\alpha w/2)-J_0(2\pi+\alpha w/2)]$
	0	$\lvert x\rvert<s/2$, $\lvert x\rvert>s/2+w$	

(a)

(b)

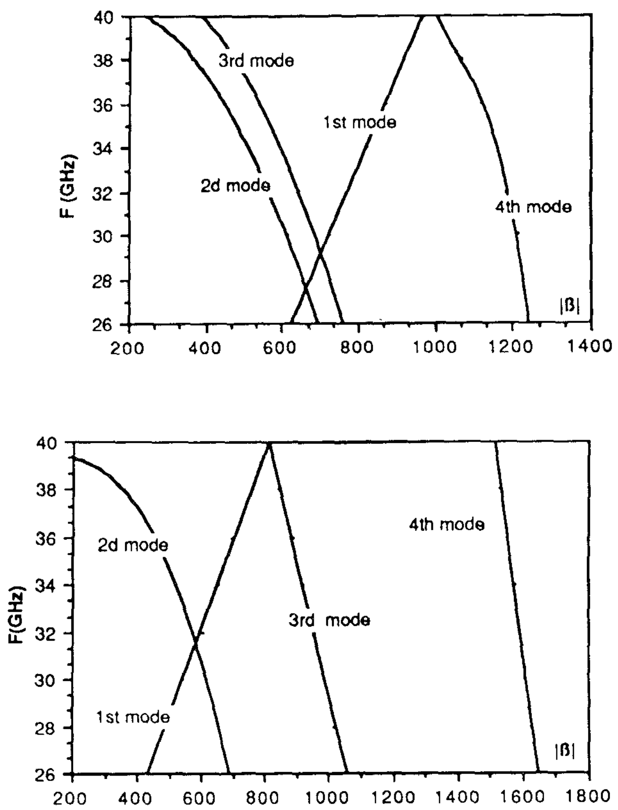


Fig. 3. Dispersion characteristics of four eigenmodes in coupled unilateral finlines (a) Odd-mode excitation. (b) Even-mode excitation.

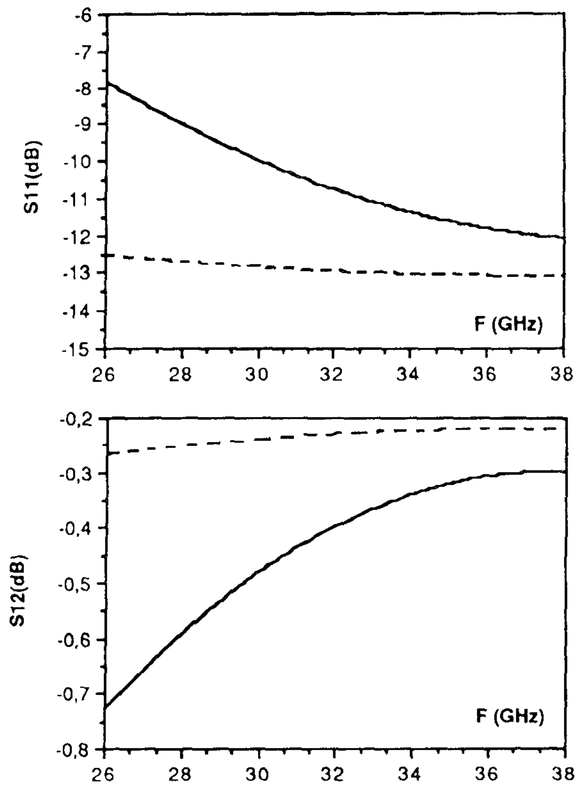


Fig 4 Reflection and transmission coefficients for —— the coplanar-slot transition for the even-mode excitation and ---- the equivalent step discontinuity in a unilateral finline technology

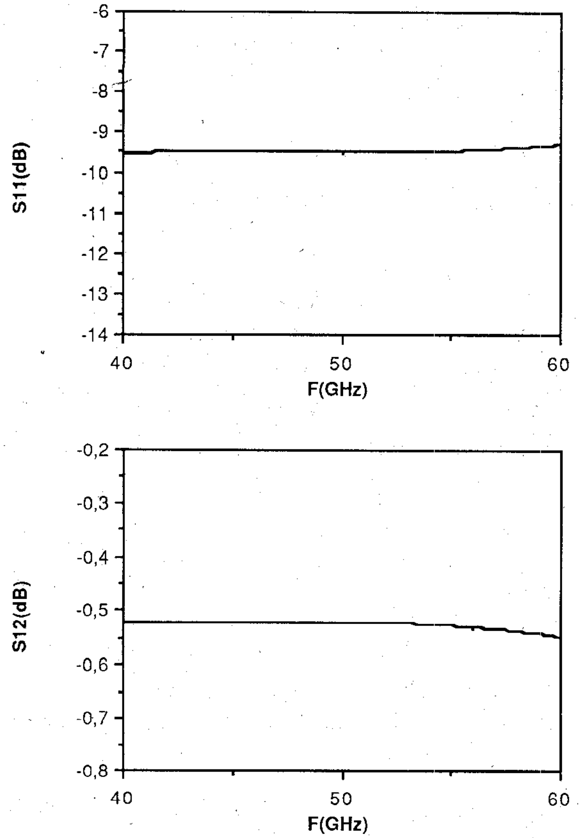


Fig. 5. Reflection and transmission coefficients for the even-mode excitation of a coplanar–slot transition operating between 40 and 60 GHz ($A = 4.775$ mm, $B = 2.388$ mm, $\epsilon_r = 2.22$ and $h_2 = 0.254$ mm).

IV. Conclusions

The spectral-domain technique was shown to be very efficient for calculating the dominant and higher order modes in coupled unilateral finlines in even- and odd-modes excitations. This technique, when combined with the direct modal analysis, is capable of determining the scattering matrix parameters of the dominant mode for the coplanar–slot transition in a unilateral technology for the odd-mode excitation and for the unavoidable even-mode excitation of the junction.

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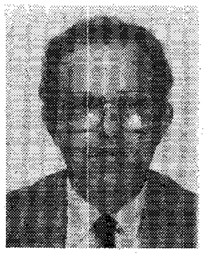
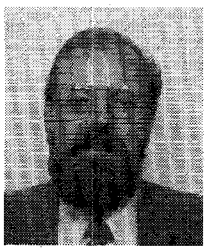
Odile Picon was born in Paris, France on March 31, 1951. She received the "agregation of Physic," from the Ecole Normale Supérieure de Fontenay-aux Roses, France in 1976 and the doctor degree (Doctorat de 3ème cycle) in external geophysic in 1980 from the University of Orsay, Fance.

Since 1982, she has been a research engineer in the Space and Radioelectric Transmission Division of the Centre National d'Etudes des Télécommunications. Her research work deals with electromagnetic theory, in particular with finline discontinuity analysis.



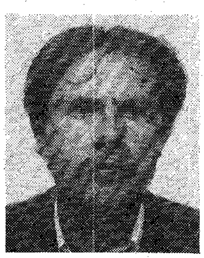
Victor Fouad Hanna (SM'87) was born in Cairo, Egypt, in 1944. He received the B.Sc. degree (honors) in electronic engineering from Cairo University in 1965 and the M.Sc. degree in microwave engineering from Alexandria University, Alexandria, Egypt, in 1969. He received the D.Sc. degree [Doctorat ès Sciences Physiques (doctorat d'Etat)] from l'Institut National Polytechnique (I.N.P.), Toulouse, France, in 1975.

He was a research assistant in the National Research Center, Cairo, Egypt, and the microwave laboratory of the I.N.P., Toulouse, France, from 1965 to 1970 and 1970 to 1975, respectively. From 1975 to 1979 he was a researcher in the Electrical and Electronics Engineering Laboratory, National Research Center, Cairo, Egypt, engaged in research in the field of microwave theory and techniques and microwave solid-state devices. Since 1979, he has been with the Centre National d'Etudes des Télécommunications, France, in its Space and Radioelectric Transmission Division, where he is now responsible of the millimetric microelectronics section. Dr. Fouad Hanna has authored or coauthored 48 technical papers. His current research interests deal with electromagnetic theory, numerical methods for solving field problems, and the characterisation of microstrip-like transmission lines and millimeter-wave transmission lines.



Jacques Citerne was born in Arras, France, on October 5, 1945. He received the doctoral degree in physics in 1978 from the Technical University of Lille, France.

From 1978 to 1980, he led the Propagation and Circuit Group of the Microwave and Semiconductor Center at the Technical University of Lille. Since 1980, he has been Professor of Electrical Engineering at the National Institute of Applied Sciences (INSA) at Rennes, France, where he created a Microwave Laboratory, which is now supported by the French National Center of Scientific Research (UA CNRS 834). The laboratory is currently working on various problems related to microwave integrated circuitry in centimeter and millimeter waves, with a special stress on the role of discontinuities in the design of open and closed passive components.



Jean-Paul Lefevre was born in Paris, France in 1961. He received the Electronical Engineering Diplome from the Ecole Spéciale de Mécanique et Electricité, Paris, in 1984, and the Higher Studies Certificate (C.E.S.) in telecommunications engineering from the Ecole Nationale Supérieure des Télécommunications, Paris, in 1986.

Since September 1986, he has been with the Division Systèmes Electroniques of Thomson-CSF in Bagneux, France.